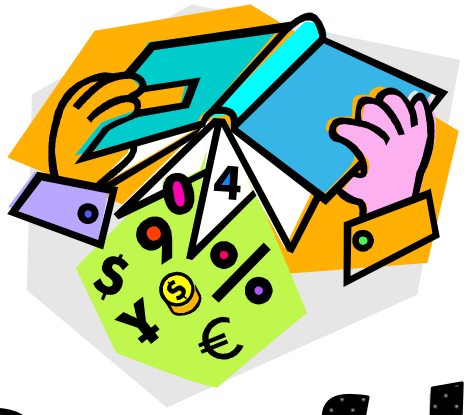
# Mathematics:



# "A Story of Units"

# Parent Handbook

## Grade 3

## Module 3

Grade 3 • Module 3

Multiplication and Division with Units of 0, 1, 6–9, and Multiples of 10

OVERVIEW

This 25-day module builds directly on students' work with multiplication and division in Module 1. By this point, Module 1 instruction coupled with fluency practice in Module 2 has students well on their way to meeting the Grade 3 fluency expectation for multiplying and dividing within 100. Module 3 extends the study of factors from 2, 3, 4, 5, and 10 to include all units from 0 to 10, as well as multiples of 10 within 100. Similar to the organization of Module 1, the introduction of new factors in Module 3 spreads across topics. This allows students to build fluency with facts involving a particular unit before moving on. The factors are sequenced to facilitate systematic instruction with increasingly sophisticated strategies and patterns.

Topic A begins by revisiting the commutative property. Students study familiar facts from Module 1 to identify known facts using units of 6, 7, 8, and 9. They realize that they already know more than half of their facts by recognizing, for example, that if they know 2 × 8, they also know 8 × 2 through commutativity. This begins a study of arithmetic patterns that becomes an increasingly prominent theme in the module. The subsequent lesson carries this study a step further; students apply the commutative property to relate 5 × 8 and 8 × 5, and then add one more group of 8 to solve 6 × 8 and, by extension, 8 × 6. The final lesson in this topic builds fluency with familiar multiplication and division facts, preparing students for the work ahead by introducing the use of a letter to represent the unknown in various positions.

Topic B introduces units of 6 and 7, factors that are well suited to Level 2 skip-counting strategies and to the Level 3 distributive property strategy, already familiar from Module 1. Students learn to compose up to, then over the next decade. For example, to solve a fact using units of 7 they might count 7, 14, and then mentally add 14 + 6 + 1 to make 21. This strategy previews the associative property using addition and illuminates arithmetic patterns as students apply count-bys to solve problems. In the next lesson, students apply the distributive property (familiar from Module 1) as a strategy to multiply and divide. They decompose larger unknown facts into smaller known facts to solve. For example, 48 ÷ 6 becomes (30 ÷ 6) + (18 ÷ 6), or 5 + 3. Topic B's final lesson emphasizes word problems, providing opportunities to analyze and model. Students apply the skill of using a letter to represent the unknown in various positions within multiplication and division problems .

Topic C anticipates the formal introduction of the associative property with a lesson on making use of structure to problem solve. Students learn the conventional order for performing operations when parentheses are and are not present in an equation. With this knowledge in place, the associative property emerges in the next lessons as a strategy to multiply using units up to 8 . Units of 6 and 8 are particularly useful for presenting this Level 3 strategy. Rewriting 6 as 2 × 3 or 8 as 2 × 4 makes shifts in grouping readily apparent (see example below), and also utilizes familiar factors 2, 3, and 4 as students learn the new material. The following strategy may be used to solve a problem like 8 × 5:

$8 \times 5 = (4 \times 2) \times 5$

$8 \times 5 = 4 \times (2 \times 5)$

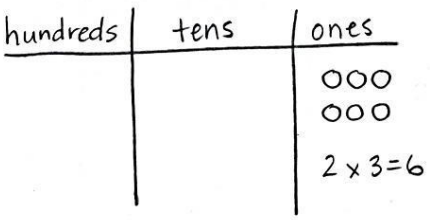
$8 \times 5 = 4 \times 10$

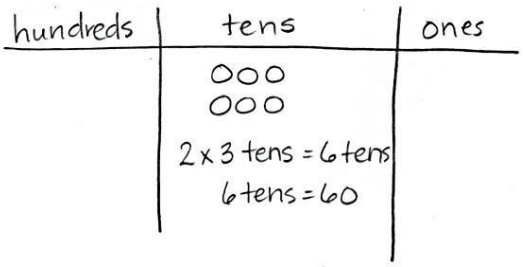
In the final lesson of Topic C, students relate division using units up to 8 with multiplication. They understand division as both a quantity divided into equal groups and an unknown factor problem for which—given the large size of units—skip-counting to solve can be more efficient than dividing.

Topic D introduces units of 9 over three days, exploring a variety of arithmetic patterns that become engaging strategies for quickly learning facts with automaticity. Nines are placed late in the module so that students have enough experience with multiplication and division to recognize, analyze, and apply the rich patterns found in the manipulation of these facts. As with other topics, the sequence ends with interpreting the unknown factor to solve multiplication and division problems.

In Topic E, students begin by working with facts using units of 0 and 1. From a procedural standpoint, these are simple facts that require little time for students to master; however, understanding the concept of nothing (zero) is among the more complex, particularly as it relates to division. This unique combination of simple and complex explains the late introduction of 0 and 1 in the sequence of factors. Students study the results of multiplying and dividing with those units to identify relationships and patterns. The topic closes with a lesson devoted to two-step problems involving all four operations. In this lesson, students work with equations involving unknown quantities and apply the rounding skills learned in Module 2 to make estimations that help them assess the reasonableness of their solutions.

In Topic F, students multiply by multiples of 10. To solve a fact like 2 × 30, they first model the basic fact 2 × 3 on the place value chart. Place value understanding helps them to notice that the product shifts one place value to the left when multiplied by 10: 2 × 3 tens can be found by simply locating the same basic fact in the tens column.




In the subsequent lesson, place value understanding becomes more abstract as students model place value strategies using the associative property. 2 × 30 = 2 × (3 × 10) = (2 × 3) × 10. The final lesson focuses on solving two-step word problems involving multiples of 10 and equations with unknown quantities. As in Lesson 18, students estimate to assess the reasonableness of their solutions.

## Terminology

### New or Recently Introduced Terms

- Even, odd (number)
- Multiple (specifically with reference to naming multiples of 9 and 10, e.g., 20, 30, 40, etc.)
- Multiplier (the factor representing the number of units)
- Product (the quantity resulting from multiplying two or more numbers together)

### Familiar Terms and Symbols

- Array (a set of numbers or objects that follow a specific pattern)
- Commutative Property (e.g., 2 x 3 = 3 x 2)
- Distribute (with reference to the distributive property; e.g., in 12 x 3 = (10 x 3) + (2 x 3), the 3 is multiplier for each part of the decomposition)
- Divide, division (partitioning a total into equal groups to show how many equal groups add up to a specific number, e.g., 15 ÷ 5 = 3)
- Equal groups (with reference to multiplication and division; one factor is the number of objects in a group and the other is a multiplier that indicates the number of groups)
- Equation (a statement that two expressions are equal, e.g., 3 x 4 = 12)
- Factors (numbers that are multiplied to obtain a product)
- Multiply, multiplication (an operation showing how many times a number is added to itself, e.g., 5 x 3 = 15)
- Number bond (model used to show part-part-whole relationships)
- Ones, twos, threes, etc. (units of one, two, or three)
- Parentheses (the symbols ( ) used around a fact or numbers within an equation)
- Quotient (the answer when one number is divided by another)
- Row, column (in reference to rectangular arrays)
- Tape diagram (a method for modeling problems)
- Unit (one segment of a partitioned tape diagram)
- Unknown (the missing factor or quantity in multiplication or division)
- Value (how much)

## Lesson 1

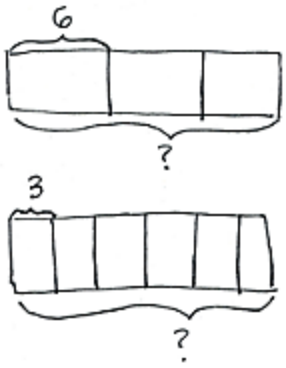
Objective: Study commutativity to find known facts of 6, 7, 8, and 9.

To show that 3 × 6 and 6 × 3 equal the same amount,

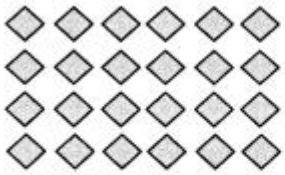
we can write 3 × 6 = 6 × 3

or

3 sixes = 6 threes




2. Use the array to write 2 different multiplication sentences.



24 = 4 × 6

24 = 6 × 4

## Lesson 2

Objective: Apply the distributive and commutative properties to relate multiplication facts 5 × n + n to 6 × n and n × 6 where n is the size of the unit.

35 is the total of 5 sevens, and 7 is the total of 1 seven.

The dots show 6 sevens broken into 5 sevens and 1 seven, because we know those facts and they're easy!


3. An author writes 9 pages of her book each week. How many pages does she write in 7 weeks? Use a five fact to solve.

5 nines

5 × 9 = 45

2 nines

2 × 9 = 18

7 × 9 = (5 × 9) + (2 × 9)

= 45 + 18

43 2 20

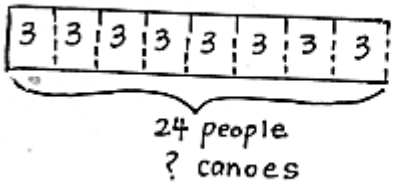
= 63

She writes 63 pages in 7 weeks.

## Lesson 3

Objective: Multiply and divide with familiar facts using a letter to represent the unknown.

**Problem 1: Use a letter to represent the unknown in multiplication.**

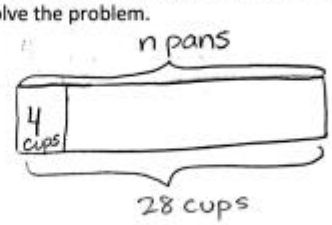



$3 \times ? = 24$
$? = 8$
The people use 8 canoes.

3. Miss Potts used a total of 28 cups of flour to bake some bread. She put 4 cups of flour in each pan. How many pans of bread did she bake? Represent the problem using multiplication and division sentences and a letter for the unknown. Then solve the problem.

$4 \times n = 28$
$28 \div 4 = n$
$n = 7$

Miss Potts made 7 pans of bread.




## Lesson 4

Objective: Count by units of 6 to multiply and divide using number bonds to decompose.

2. Count by six to fill in the blanks below.

6, 12, 18, 24

Complete the multiplication equation that represents the final number in your count-by.

$6 \times 4 = 24$

Complete the division equation that represents your count-by.

$24 \div 6 = 4$

$12 + 6 = 18$ (10, 2) — $2 + 6 = 8$, $10 + 8 = 18$

$18 + 6 = 24$ (2, 4) — $18 + 2 = 20$, $20 + 4 = 24$

$24 + 6 = 30$ (20, 4) — $4 + 6 = 10$, $20 + 10 = 30$

$30 + 6 = 36$

$36 + 6 = 42$ (4, 2) — $36 + 4 = 40$, $40 + 2 = 42$

$42 + 6 = 48$ (40, 2) — $2 + 6 = 8$, $40 + 8 = 48$

$48 + 6 = 54$ (2, 4) — $48 + 2 = 50$, $50 + 4 = 54$

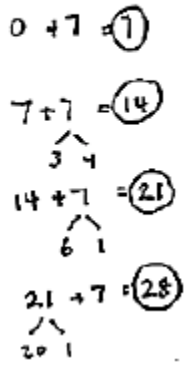
9. $54 + 6 = 60$ (50, 4) — $4 + 6 = 10$, $50 + 10 = 60$

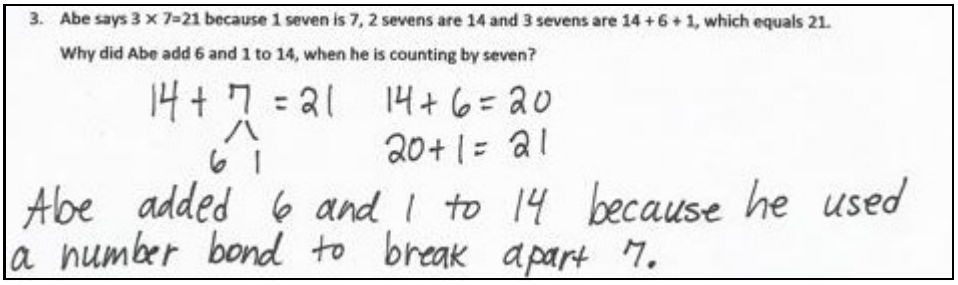
## Lesson 5

Objective: Count by units of 7 to multiply and divide using number bonds to decompose.

*Decomposing helps us to make a ten and add quickly. This helps us find unknown products. When we know a near multiplication fact we can use it to count on.*

Count by seven 4 times:

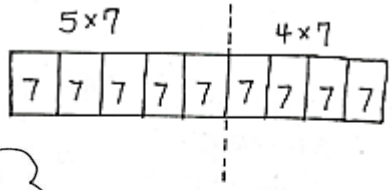


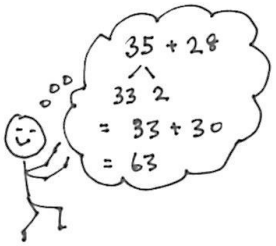




## Lesson 6

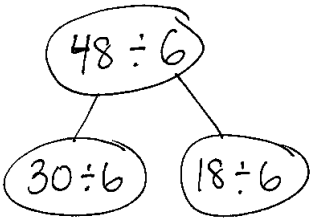
Objective: Use the distributive property as a strategy to multiply and divide using units of 6 and 7.

Break apart a bigger fact into 2 easy facts This makes it easier to solve because we just add the products of the 2 easy facts.




9×7 =
(5×7) + (4×7) =
35 + 28 = 63
9×7 = 63



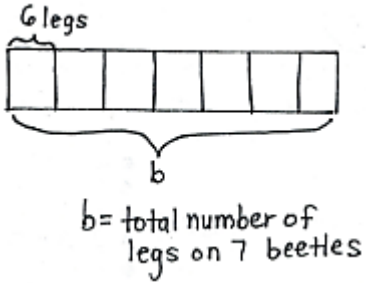

48 ÷ 6 = (30 ÷ 6) + (18 ÷ 6)
= 5 + 3
= 8

We broke apart 48 ÷ 6 into two smaller division expressions. 30 makes a good breaking point because 30 ÷ 6 is an easy fives fact.
48 ÷ 6 = (30 ÷ 6) + (18 ÷ 6)

## Lesson 7

Objective: Interpret the unknown in multiplication and division to model and solve problems using units of 6 and 7.

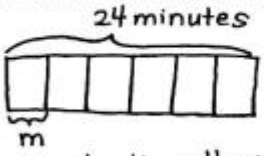
Thad sees 7 beetles when he weeds his garden. Each beetle has 6 legs. How many legs are there on all 7 beetles?




$7 \times 6 = b$

$b = 42$

b. Henry spends 24 minutes practicing 6 different basketball drills. He spends the same amount of time on each drill. How much time does Henry spend on each drill?




$24 \div 6 = m$

$m = 4$

Henry spends 4 minutes on each drill.

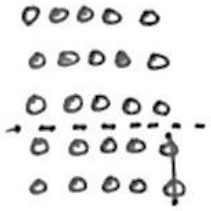
## Lesson 8

Objective: Understand the function of parentheses and apply to solving problems.

We can use parentheses in our equation to show what to do first.

First we divide (10÷5)

Then we subtract 25-2 = 23

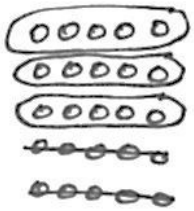


$25 - (10 \div 5) = 23$

2

Here we subtracted 25-10 first.

Then we divided 15 ÷ 5 = 3



$(25 - 10) \div 5 = 3$

15

Placing the parenthesis around different expressions in an equation can yield very different results.

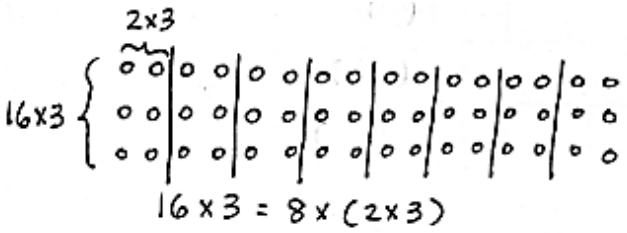
## Lesson 9

Objective: Model the associative property as a strategy to multiply.

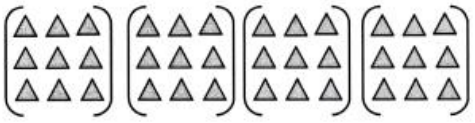
8, and 2 are much friendlier factors than 16.

*Rewrite 16 as 8 × 2. 16 x 3= 8 x (2 x 3)*

*We put the 2 and 3 in parentheses because they are easy to multiply.*




a) 3 × 12 = 36



b) (3 × 3) × 4
= 9 × 4
= 36

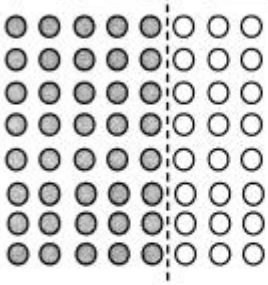
16 x 2 = (8 x 2) x 2
= 8 x (2 x 2)
= 8 x 4
= 32

## Lesson 10

Objective: Use the distributive property as a strategy to multiply and divide.

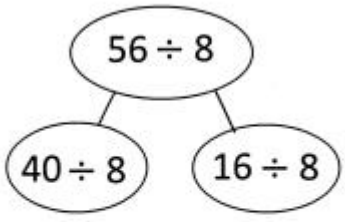
a) 8 × 8 = 64

(8 × 5) = 40 | (8 × 3) = 24



8 × 8 = 8 × (5 + 3)
= (8 × 5) + (8 × 3)
= 40 + 24
= 64

2. Break apart and distribute to solve 56 ÷ 8.



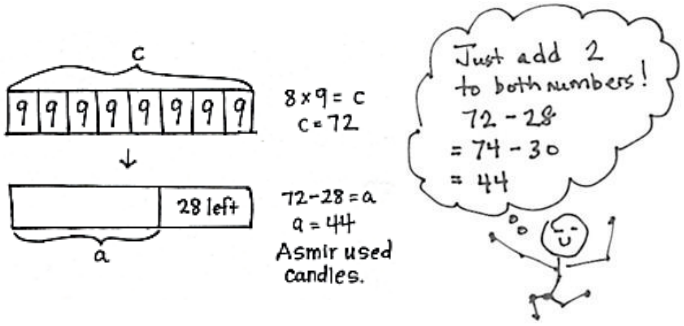

56 ÷ 8 = (40 ÷ 8) + (16 ÷ 8)
= 5 + 2
= 7

## Lesson 11

Objective: Interpret the unknown in multiplication and division to model and solve problems.

**Problem 1: Interpret the unknown in multiplication.**

Asmir buys 8 boxes of 9 candles for his dad's birthday. After putting some candles on the cake, there are 28 candles left. How many candles does Asmir use?

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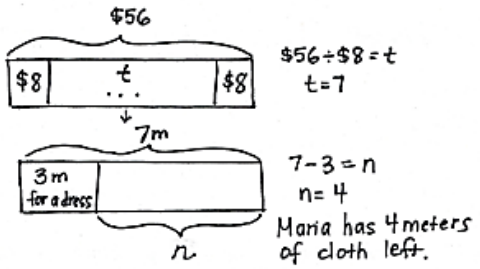
**Problem 2: Interpret the unknown in division.**

The fabric store sells 1 meter of cloth for $8. Maria buys some cloth that costs a total of $56. She then uses 3 meters to sew a dress. How many meters of cloth does she have left?

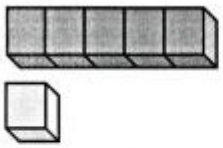



## Lesson 12

Objective: Apply the distributive property and the fact 9 = 10 – 1 as a strategy to multiply.

1. Each  has a value of **9**. Find the value of each row. Then add the rows to find the total.

a. **6 × 9 =** 54



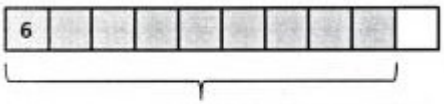
5 × 9 = 45



1 × 9 = 9

**6 × 9** = (5 + 1) × 9
= (5 × 9) + (1 × 9)
= 45 + 9
= 54

a. **9 × 6 =** 54




**9 sixes** = 10 sixes – 1 six
= 60 − 6
= 54

## Lesson 13

Objective: Identify and use arithmetic patterns to multiply.

- Look at each place value and identify how they change.

1. a. Skip-count by nine.

9, 18, 27, 36, 45, 54, 63, 72, 81, 90

b. Look at the **tens** place in the count-by. What is the pattern?

The tens place increases by 1.

c. Look at the **ones** place in the count-by. What is the pattern?

The ones place decreases by 1.

(9)

9+10=19

(18) 9 is 1 less than 10 so:
9+9=18

18+10=28

(27) 9 is 1 less than 10 so:
18+9=27

27+10=37

(36) 9 is 1 less than 10 so:
27+9=36

36+10=46

(45) 9 is 1 less than 10 so:
36+9=45

45+10=55

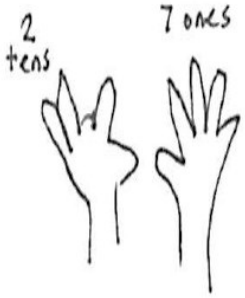
(54) 9 is 1 less than 10 so:
45+9=54

## Lesson 14

Objective: Identify and use arithmetic patterns to multiply.

**Finger Trick**

To solve a nines fact, lower the finger that matches the number of nines.




3x9 = 2 tens 7 ones
= 27

**Unit Form**

To get 1 nine, we subtract 1 from a ten.

To solve 9 x3 think about 3 tens. You have to take 1 away from each 10 to make it 3 nines. So you subtract 3

30 - 3 = 27 = 9 x 3

**Sum of the Digits**

If the sum of the digits in your product equals 9 your multiplication is correct.

9 x 3 = 27  5 x 9 = 45

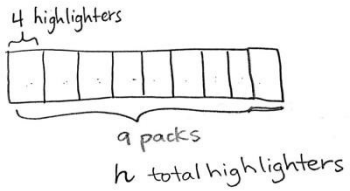
2+ 7 = 9  4+5 = 9

# Lesson 15

Objective: Interpret the unknown in multiplication and division to model and solve problems.

**Problem 1: Interpret the unknown in multiplication.**

Write or project the following problem: Ada buys 9 packs of highlighters with 4 in each pack. After giving 1 highlighter to each classmate, she has 17 left. How many highlighters did Ada give away?




$9 \times 4 = h$

$h = 36$

$36 - 17 = g$

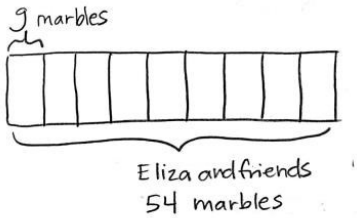
$g = 19$

**Problem 2: Interpret the unknown in division.**

Write the following problem: Eliza finds a bag of 72 marbles and runs to share them with 8 of her friends. She's so excited that she drops the bag and loses 18 marbles. How many marbles will Eliza and each of her friends get?

$72 - 18 = m$

$m = 54$



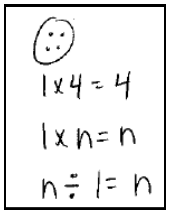

$54 \div 9 = g$

$g = 6$ marbles

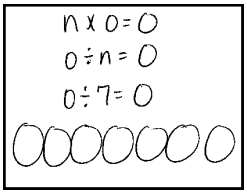
# Lesson 16

Objective: Reason about and explain arithmetic patterns using units of 0 and 1 as they relate to multiplication and division.

**Multiplying and Dividing by 1**

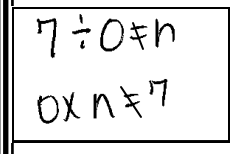



Let *n* equal the number of dots in each group. What is 1 times *n* dots? It's *n*, because the number of dots in each group is the same as the total number of dots . Any number times 1 equals that number, any number divided by 1 equals that number, and any number divided by itself equals 1.



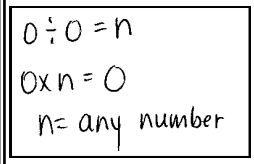

**Dividing Zero and Multiplying by Zero** Any number times 0 equals 0. Zero divided by a number equals 0.

**Dividing by Zero**




$7 \div 0 = n$ The related multiplication fact is $0 \times n = 7$. This does not work. Any number times 0 equals 0. There's no value for *n* that would make a true multiplication sentence, and the same is true for the division equation. You cannot divide by zero.

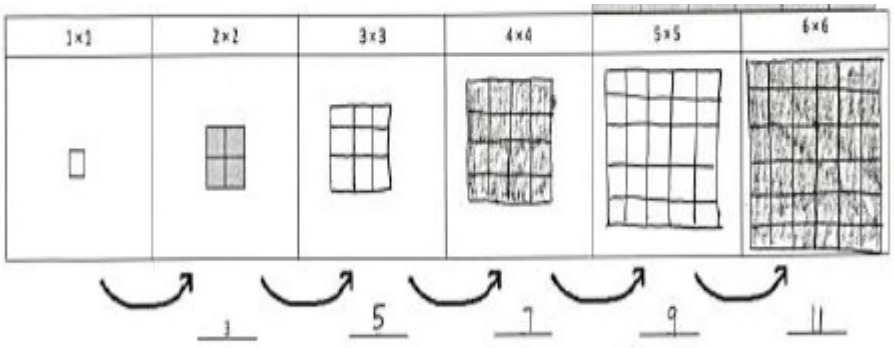
**Dividing Zero**




*n* can be any number in the multiplication equation, and the same is true for the division equation.

# Lesson 17

Objective: Identify patterns in multiplication and division facts using the multiplication table.




n × n is the sum of the first n odd numbers. So...

4 x 4 = 16

Add up the first 4 odd numbers to get the same result.

1 + 3 + 5 + 7 = 16

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 1×1 1 | 2×1 2 | 3×1 3 | 4×1 4 | 5×1 5 | 6×1 6 | 7×1 7 | 8×1 8 |
| 1×2 2 | 2×2 4 | 3×2 6 | 4×2 8 | 5×2 10 | 6×2 12 | 7×2 14 | 8×2 16 |
| 1×3 3 | 2×3 6 | 3×3 9 | 4×3 12 | 5×3 15 | 6×3 18 | 7×3 21 | 8×3 24 |
| 1×4 4 | 2×4 8 | 3×4 12 | 4×4 16 | 5×4 20 | 6×4 24 | 7×4 28 | 8×4 32 |
| 1×5 5 | 2×5 10 | 3×5 15 | 4×5 20 | 5×5 25 | 6×5 30 | 7×5 35 | 8×5 40 |
| 1×6 6 | 2×6 12 | 3×6 18 | 4×6 24 | 5×6 30 | 6×6 36 | 7×6 42 | 8×6 48 |
| 1×7 7 | 2×7 14 | 3×7 21 | 4×7 28 | 5×7 35 | 6×7 42 | 7×7 49 | 8×7 56 |
| 1×8 8 | 2×8 16 | 3×8 24 | 4×8 32 | 5×8 40 | 6×8 48 | 7×8 56 | 8×8 64 |

Color all the squares with even products orange. Can an even product ever have an odd factor?
Yes, because odd times even equals even.

Can an odd product ever have an even factor?
No, because odd products only happen with 2 odd factors.

Everyone knows that 7 x 4 = (5 x 4) + (2 x 4). Explain how this is shown in the table.
The table shows 5x4 = 20 and 2x4 = 8. So, 20 + 8 = 28, which is the product of 7x4

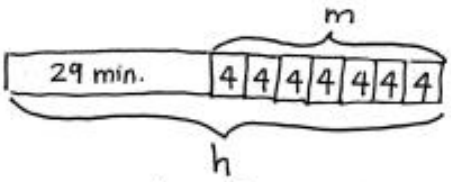
Use what you know to find the product of 7 x 16 or 8 sevens + 8 sevens.

7 x 16 = (7x8) + (7x8)
= 56 + 56
= 112

# Lesson 18

Objective: Solve two-step word problems involving all four operations and assess

2. Julio spends 29 minutes doing his spelling homework. He then completes each math problem in 4 minutes. There are 7 math problems. How many minutes does Julio spend on his homework in all?




m = number of minutes spent on math homework

7 x 4 = m
m = 28 minutes

h = number of minutes spent on homework

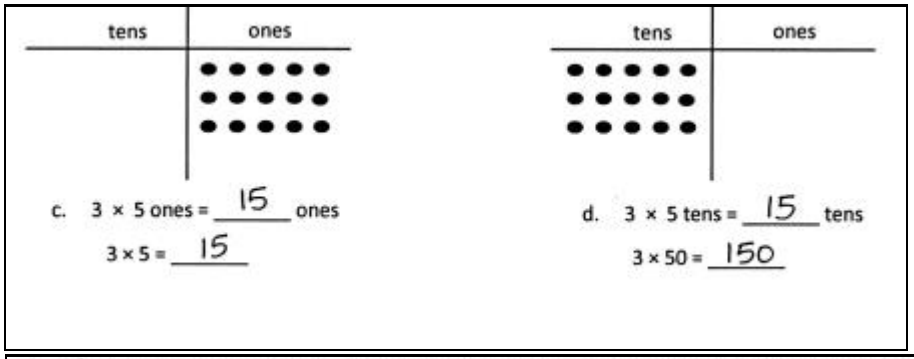
29 + 28 = h
h = 57 minutes

My answer is reasonable because 57 minutes is almost an hour and 29 and 28 are each about half an hour. Two half hours equal an hour.

Julio spends 57 minutes on his homework.

## Lesson 19

Objective: Multiply by multiples of 10 using the place value chart.



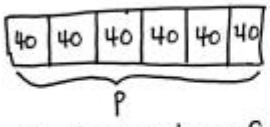

c. $3 \times 5$ ones = 15 ones

$3 \times 5 =$ 15

d. $3 \times 5$ tens = 15 tens

$3 \times 50 =$ 150

4. A bus can carry 40 passengers. Use a tape diagram to find how many passengers 6 buses can carry.




p = the number of passengers 6 buses can carry

$6 \times 40 = p$

$p = 240$

6 buses can carry 240 passengers.

## Lesson 20

Objective: Use place value strategies and the associative property $n \times (m \times 10) = (n \times m) \times 10$ (where $n$ and $m$ are less than 10) to multiply multiples of 10.

3. Gabriella solves $20 \times 4$ by thinking about $10 \times 8$. Explain her strategy.

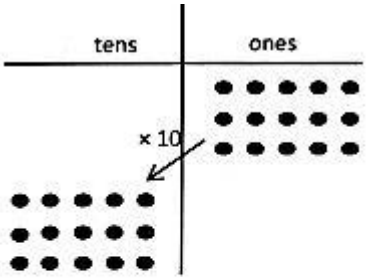
$20 \times 4 = (10 \times 2) \times 4$

$= 10 \times (2 \times 4)$

$= 10 \times 8$

$= 80$

Gabriella breaks the $20 \times 4$ into $10 \times 2 \times 4$. Then she moves the ( ) over to $2 \times 4$. This makes the problem easier to solve. Instead of thinking of the problem as $20 \times 4$, she can solve by thinking of an easier fact, $10 \times 8$.




c) $(3 \times 5) \times 10$

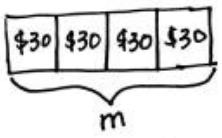
$=$ ( 15 ones ) $\times 10$

$=$ 150

## Lesson 21

Objective: Solve two-step word problems involving multiplying single-digit factors and multiples of 10.

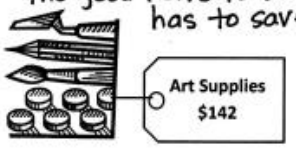
2. Lupe saves $30 each month for 4 months. Does she have enough money to buy the art supplies below? Explain why or why not.



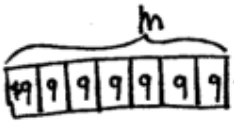

4 x $30 = m
m = $120
Lupe saved $120 in 4 months.

$142 - 120 = $22

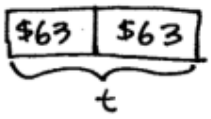
Lupe does not have enough money to buy the art supplies. She is $22 short. The good news is she just has to save for 1 more month.




6. Ezra earns $9 an hour working at a book store. She works for 7 hours each day on Mondays and Wednesdays. How much does Ezra earn each week?




m = 7 x 9
= 63




t = 63 + 63
t = 126

Ezra earns $126 each week.

It' s important to become fluent with multiplication and division facts. Quick 5-10 minute activities are essential for memorization. Here are some ways to assist your child with memorizing basic facts:

- Flash Cards
  - ◊ both you and your child should say the fact aloud
  - ◊ begin learning them in order
- Skip counting up and down. Try beginning at different starting points.
  - ◊ ie: 3, 6, 9, 12– 9, 6 ,3 16, 20, 24, 28, 32-28, 24, 20, 16
- Have quick routine math talks in the car, store, and anywhere that seems appropriate.
- Computer Aides such as xtramath.org